



Anomalous pinning fields in itinerant helical magnets: Screening of the quasiparticle interaction

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The spin-orbit interaction strength g_{so} in helical magnets determines both the pitch wave number q and the critical field H_{c1} where the helix aligns with an external magnetic field. Within a standard Landau-Ginzburg-Wilson (LGW) theory, a determination of g_{so} in MnSi and FeGe from these two observables yields values that differ by a factor of 20. This discrepancy is remedied by considering the fermionic theory underlying the LGW theory and in particular the effects of screening on the effective electron-electron interaction that results from an exchange of helical fluctuations.

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Chiral itinerant ferromagnets such as MnSi (Refs. 1 and 2) and FeGe (Ref. 3) have recently attracted considerable attention. Both of these systems crystallize in the cubic $B20$ structure, which lacks inversion symmetry, and as a result spin-orbit coupling effects are important for the magnetic properties. They both exhibit spiral or helimagnetic spin order at low temperatures (below a critical temperature T_c of about 28.5 K at ambient pressure in MnSi, and about 279 K in FeGe, respectively), which is believed to be generated by a Dzyaloshinsky-Moriya term^{4,5} in the free energy. The pitch wavelength of the helix is large compared to a microscopic length scale; this reflects the weakness of the spin-orbit interaction. The pitch wave vector is $q=0.035 \text{ \AA}^{-1}$ in MnSi (Ref. 6) and $q \approx 0.009 \text{ \AA}^{-1}$ in FeGe.³ In other parts of the phase diagram in MnSi, striking non-Fermi-liquid behavior has been observed in low-temperature transport measurements.⁷

In an isotropic electron system there would be no preferred direction for the pitch vector of the helix. In real materials, the underlying crystal lattice pins the helix. The terms in the free energy that cause this pinning are of higher order in the spin-orbit interaction and hence represent an energy scale that is even weaker than those that lead to the formation of the helix. In MnSi, the pinning is in the (1,1,1) direction (or equivalent); in FeGe, it is in the (1,0,0) direction (or equivalent) close to the transition, and in the (1,1,1) direction at lower temperatures. An external magnetic field makes it energetically favorable for the helix to align with the field, and this competes with the crystal-field effects. As a result, upon applying a magnetic field in, say, the (0,0,1) direction to MnSi in the helical phase, the pitch vector rotates away from (1,1,1) until it aligns with the field direction at a critical field strength $H=H_{c1}$. This field strength marks the boundary of the so-called conical phase, which is characterized by a homogeneous magnetization superimposed on the helix that is aligned with the field. Upon further increasing the field, the homogeneous component of the magnetization increases, and the amplitude of the helix continuously decreases until it disappears at a second critical field, H_{c2} , where the system enters a field-polarized phase with a homogeneous magnetization.

The above considerations make it clear that one can ob-

tain a measure of the spin-orbit interaction strength from measuring either the pitch wave number or the critical field H_{c1} . It is a puzzling, but overlooked, fact that interpreting the results within the existing theoretical framework yields values for the spin-orbit interaction strength that differ by a factor of about 25. It is the purpose of the present Rapid Communication to resolve this discrepancy.

In order to frame our discussion of these various effects, let us consider the Landau-Ginzburg-Wilson (LGW) theory that has been commonly used to describe helical magnets.⁸ If the phase transition is either continuous or weakly first order, then the classical behavior of the system close to the transition can be described by an action

$$S = S_0 + S_c + S_{cf} + S_H. \quad (1a)$$

Here S_0 is the usual action for a classical Heisenberg ferromagnet,⁹

$$S_0 = \int dx \left\{ \frac{r_0}{2} \mathbf{M}^2(\mathbf{x}) + \frac{a}{2} [\nabla \mathbf{M}(\mathbf{x})]^2 + \frac{u}{4} \mathbf{M}^4(\mathbf{x}) \right\}. \quad (1b)$$

Here $\mathbf{M}(\mathbf{x})$ is the three-component order parameter whose expectation value is proportional to the magnetization, r_0 is the bare distance from the critical point, and a and u are parameters that depend on the microscopic details of the system. $(\nabla \mathbf{M})^2$ is a shorthand notation for $\sum_{i,j} \partial_i M_j \partial^i M^j$. S_0 is invariant under separate rotations in order-parameter space and real space.

S_c is the leading chiral term induced by the spin-orbit interaction,^{4,5}

$$S_c = \frac{c}{2} \int dx \mathbf{M}(\mathbf{x}) \cdot [\nabla \times \mathbf{M}(\mathbf{x})]. \quad (1c)$$

The coupling constant c is proportional to the dimensionless spin-orbit interaction strength g_{so} , and on dimensional grounds we have $c = a k_F g_{so}$. Note that S_c is still invariant under joint rotations in order-parameter space and real space but not under spatial inversions. This term can therefore be present only in systems that are not inversion invariant. The chiral nature of the curl produces the helical ground state,

and the handedness of the helix depends on the sign of c . We assume $c > 0$ without loss of generality.

S_{cf} is the largest term that describes the crystal-field effects that couple the magnetization to the underlying lattice. For a cubic lattice, a representative contribution to S_{cf} reads⁸

$$S_{cf} = b \int dx \sum_i \left(\frac{\partial M_i}{\partial x_i} \right)^2. \quad (1d)$$

The coupling constant b is quadratic in g_{so} and given by $b = a' g_{so}^2$ with $|a'| \approx a$. [Here and in what follows we ignore factors of $O(1)$.] S_{cf} breaks the rotational invariance and is responsible for pinning the helix. The direction of the pinning depends on the sign of b .

Finally, we have a term that couples an external magnetic field \mathbf{H} to the magnetization:

$$S_H = \int dx \mathbf{H}(\mathbf{x}) \cdot \mathbf{M}(\mathbf{x}). \quad (1e)$$

In the absence of both an external field and any coupling to the underlying lattice, it is easy to see that Eqs. (1b) and (1c) lead to a helical ground state:

$$\mathbf{M}(\mathbf{x}) = m_1 [\hat{e}_1 \cos(\mathbf{q} \cdot \mathbf{x}) + \hat{e}_2 \sin(\mathbf{q} \cdot \mathbf{x})]. \quad (2)$$

Here the unit vectors \hat{e}_1 , \hat{e}_2 , and $\hat{\mathbf{q}} = \mathbf{q}/|\mathbf{q}|$ form a right-handed *dreibein*. The amplitude of the helix is given by $m_1 = \sqrt{-r/u}$, with $r = r_0 - aq^2$. The pitch vector \mathbf{q} points in an arbitrary but fixed direction, and in a mean-field approximation its modulus is given by $q = c/2a + O(g_{so}^2)$. q is small compared to the Fermi wave number k_F by virtue of $g_{so} \ll 1$. In MnSi, $k_F \approx 3.6 \text{ \AA}^{-1}$,¹⁰ so $q/k_F \approx 0.01$. Assuming the same value for k_F in FeGe, we have $q/k_F \approx 0.0025$. The value of g_{so} , which is equal to q/k_F within a factor of two,¹¹ is thus

$$g_{so} \approx \begin{cases} 0.01 & (\text{MnSi}) \\ 0.0025 & (\text{FeGe}). \end{cases} \quad (3)$$

As discussed above, a magnetic field tends to align the helix away from the pinning direction that is ultimately determined by the spin-orbit interaction, and hence the magnitude of the field necessary to depin the helix provides another estimate for g_{so} . In MnSi, the best studied helimagnet, the helix in zero field is pinned in the (1,1,1) direction, which implies that the coefficient b in Eq. (1d) is negative ($b > 0$ leads to pinning in the (1,0,0) direction).⁸ At ambient pressure and not too close to the transition temperature, the experimental value for the field H_{c1} defined above, where the pitch vector \mathbf{q} aligns with the field direction, is $H_{c1} \approx 0.1 \text{ T}$.¹² In the same region, the experimental value for the field H_{c2} , where the helix vanishes, varies between 0.4 and 0.55 T. Together with the corresponding experimental results for FeGe,³ we thus have a ratio

$$\Delta_{\text{exp}} \equiv H_{c1}/H_{c2} \approx \begin{cases} 0.2 & (\text{MnSi}) \\ 0.1 & (\text{FeGe}). \end{cases} \quad (4)$$

Comparing these experimental results with theoretical estimates leads to a puzzle. Equations (1) can be analyzed in detail to yield the critical fields H_{c1} and H_{c2} ,^{13,14} but for our present purposes the following simple considerations suffice.

H_{c1} is roughly determined by the magnetic energy, given by S_H , being equal to the pinning energy, which is given by S_{cf} . H_{c2} is roughly determined by the magnetic energy being equal to the chiral energy, which is given by S_c . For a homogeneous magnetic field, the coupling in S_H is to the homogeneous magnetization, $m_0 \equiv \int dx \mathbf{M}(\mathbf{x}) = \chi \mathbf{H}$, with χ as the homogeneous magnetic susceptibility. The magnetic energy is thus of $O(H^2)$, $S_H = \chi H^2$. In Eq. (1d), the gradient squared is on the order of q^2 , and the magnetization is on the order of the amplitude of the helix, m_1 . For H_{c1} we thus obtain the estimate

$$H_{c1} \approx m_1 q \sqrt{b/\chi} \approx g_{so} m_1 q \sqrt{a/\chi}. \quad (5a)$$

Applying an analogous estimate to Eq. (1c), we obtain

$$H_{c2} \approx m_1 \sqrt{c q/\chi} = m_1 \sqrt{a k_F g_{so} q/\chi}. \quad (5b)$$

All quantities whose estimates might be questionable thus drop out of the ratio Δ . In particular, Δ is independent of the LGW coefficient a , which is given by a length scale whose nature depends on the degree of sophistication that is built into the LGW theory. We thus have a theoretical result from the bare LGW theory¹⁵

$$\Delta_{\text{theo}}^{\text{bare}} \approx \sqrt{g_{so} q/k_F} \approx g_{so}. \quad (6)$$

Comparing Eqs. (3), (4), and (6), we see that the experimental values for Δ are larger than the theoretical expectation by about a factor of 20 in MnSi and 40 in FeGe. We will now show how this discrepancy can be resolved by considering the screening of the effective electron-electron interaction that results from the exchange of helical fluctuations.

We first need to discuss the nature of the dominant fluctuations in a helical magnet. The helical ground state represents a spontaneous breaking of translational invariance and therefore leads to a Goldstone mode or helimagnon. For the LGW action written above and as a function of the wave vector \mathbf{k} for $|\mathbf{k}| \ll q$, the frequency of the helimagnon reads^{16,17}

$$\omega_0(\mathbf{k}) = \sqrt{a k_{\parallel}^2 + 2|b| \mathbf{k}_{\perp}^2/3 + a \mathbf{k}_{\perp}^4/2q^2}. \quad (7)$$

Here $\mathbf{k} = (k_{\parallel}, \mathbf{k}_{\perp})$ has been decomposed into components parallel and perpendicular to the pitch wave vector \mathbf{q} . We have assumed $b < 0$, as appropriate for MnSi,⁸ and we have neglected corrections of $O(b) = O(g_{so}^2)$ to the coefficients of k_{\parallel}^2 and \mathbf{k}_{\perp}^4 . For $b = 0$, that is, if we neglect all effects of $O(g_{so}^2)$, the helimagnon frequency squared lacks a contribution proportional to \mathbf{k}_{\perp}^2 . This is a result of rotational invariance. The crystal-field term S_{cf} in the action breaks this invariance, which leads to a mode that is still soft (because the translational invariance is still broken), but has a \mathbf{k}_{\perp}^2 term with a small prefactor. It is the generalization of the well-known magnons in ferromagnets and antiferromagnets that have a quadratic and linear dispersion relation, respectively.

The spin model described by Eqs. (1) can be understood and derived as an effective theory that results from an underlying fermionic action. The technical procedure is to single out the magnetization by either performing a Hubbard-Stratonovich decoupling of the spin-triplet interaction or by constraining the appropriate combination of fermion fields to

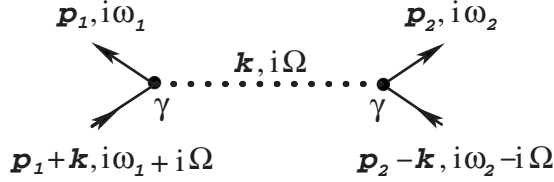


FIG. 1. The effective quasiparticle interaction due to helimagnons. Note that the vertices depend on the quasiparticle momenta in addition to the helimagnon momentum.

an auxiliary composite field whose expectation value is the magnetization.^{18,19} Integrating out the fermions then yields the spin model, with the coefficients of the LGW theory given in terms of localized fermionic correlation functions. Conversely, integrating out the magnetization yields an effective theory of electronic quasiparticles that interact via an exchange of helimagnons. This effective interaction was derived and discussed in Ref. 20. For small wave numbers, the leading contribution to the effective potential is

$$V(\mathbf{k}, i\Omega; \mathbf{p}_1, \mathbf{p}_2) = V_0 \chi(\mathbf{k}, i\Omega) \gamma(\mathbf{k}, \mathbf{p}_1) \gamma(-\mathbf{k}, \mathbf{p}_2). \quad (8a)$$

Here $V_0 = \lambda^2 q^2 / 8(m_c^*)^2$ with λ as the Stoner gap (i.e., the splitting of the two electron bands that results from the magnetization in a mean-field approximation) and m_c^* the electron effective mass. Ω denotes a bosonic Matsubara frequency, and the helimagnon susceptibility is

$$\chi(\mathbf{k}, i\Omega) = \frac{1}{2N_F} \frac{q^2}{3k_F^2} \frac{1}{\omega_0^2(\mathbf{k}) - (i\Omega)^2}, \quad (8b)$$

with N_F as the density of states at the Fermi surface. The leading contribution to the vertex function γ is given by

$$\gamma(\mathbf{k}, \mathbf{p}) = \nu (\mathbf{k}_\perp \cdot \mathbf{p}_\perp) p_\parallel / \lambda. \quad (8c)$$

The dimensionless parameter ν describes the coupling of the electrons to the lattice; generically, $\nu = O(1)$.²⁰ The effective potential is graphically depicted in Fig. 1.

Due to the singular nature of the helimagnon susceptibility the effective interaction is long ranged, and screening has a qualitative effect. The leading effect of screening is captured by the usual random-phase approximation (RPA).²¹ The screened interaction is shown in Fig. 2. If one takes the resulting fermionic theory of quasiparticles interacting via the screened effective interaction, reintroduces the magnetization, and integrates out the fermions, one can study the effect of the screening on the LGW spin model.²² For our present purposes, the most important effect is a renormalization of the coupling constant in the crystal-field term S_{cf} [Eq. (1d)]. We find

$$b \rightarrow b - |b| \nu^2 (\epsilon_F / \lambda)^2. \quad (9)$$

The value of λ in MnSi is not well known, but it is clear from the small value of T_c that ϵ_F / λ is large compared to unity. An

band structure calculation²³ yielded $\lambda \approx 3000$ K, while the Fermi temperature is $T_F \approx 150\,000$ K,^{10,24} for a ratio $\epsilon_F / \lambda \approx 50$. The effect of the screening is thus large, and the sign of the effect is important. If $b < 0$, then the bare theory yields a helix pinned in the (1,1,1) direction and the renormalization greatly enhances the coefficient and the pinning strength. If $b > 0$, then the bare theory predicts pinning in the (1,0,0) direction and the renormalization changes this to the (1,1,1) direction. This is analogous to the effect of a strong electron-phonon coupling that can trigger a structural phase transition. The pinning will thus always be in the (1,1,1) direction for systems where the transition is continuous or weakly first order. For strongly first-order transitions the LGW theory is no longer controlled, and a gradient-free cubic anisotropy in the action [i.e., a term proportional to $\sum_i M_i^4$, which we have neglected in Eqs. (1)] can invalidate this conclusion. These observations are consistent with experimental results. In MnSi, the transition is continuous or very weakly first order, and the pinning is in (1,1,1) direction everywhere in the ordered phase.² In FeGe, the transition is strongly first order, and the pinning is in (1,0,0) direction close to T_c but switches to the (1,1,1) direction at lower temperatures.³

According to Eqs. (4), H_{c1} is proportional to \sqrt{b} while H_{c2} is independent of b . The renormalized theory thus yields the following result for the ratio $\Delta = H_{c1} / H_{c2}$:

$$\Delta_{\text{theo}} \approx g_{\text{so}} \nu \epsilon_F / \lambda, \quad (10)$$

which replaces Eq. (6). With the numbers quoted above and assuming $\nu \approx 1$, we obtain values for MnSi and FeGe that are in agreement with the experimental ones given in Eq. (3) within a factor of two.

As a check, we discuss the absolute values of H_{c1} and H_{c2} . Taking into account Eq. (9), H_{c1} is given by

$$H_{c1} \approx g_{\text{so}} m_1 q \nu (\epsilon_F / \lambda) \sqrt{a / \chi}. \quad (11)$$

In the ordered phase of MnSi at ambient pressure, the susceptibility is observed to be $\chi \approx 6 \mu_B^2 / k_B T_c$,² consistent with theoretical considerations.²⁵ In a fully renormalized spin model, the gradient-squared term in the action taken at the Fermi length scale must obey $a k_F^2 m_1^2 / 2 \approx T_c$. With these estimates, we obtain from Eq. (11) $H_{c1} \approx 0.25$ T, which is the correct order of magnitude. Similarly, from Eq. (5b), $H_{c2} \approx 0.5$ T, in agreement with the experimental value. Note that without the screening that affects the value of b and is necessary to produce a ‘‘fully renormalized spin model,’’ the corresponding values would be at odds with the experimental ones even though the LGW coefficient a has been linked to the energy or length scale related to T_c rather than to the microscopic scale given by T_F . See also Ref. 15.

In summary, we have pointed out that the standard LGW theory for helical magnets leads to a large discrepancy between the strength of the spin-orbit interaction in helical magnets as determined from the pitch wave number versus

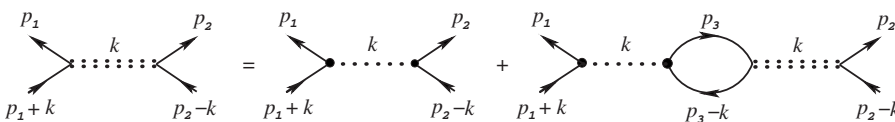


FIG. 2. Screening of the effective quasiparticle interaction.

the ratio H_{c1}/H_{c2} of the two critical fields. We have shown that a renormalization of the theory that results from the screening of the effective quasiparticle interaction resolves this puzzle and also leads to absolute values of the critical fields that are in good agreement with the experimentally observed values.

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